Simulation of Panlevé-Gullstrand black hole in thin ^{3}He-A film.

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The quasi-stationary superfluid state is constructed, which exhibits the event horizon and Hawking radiation.

Contents

A. *Introduction*

It is well known that the gravitational field can be simulated in condensed matter by the motion of the liquid: propagation of some perturbations in the moving liquid obeys the same equation as propagation of relativistic particles in the gravitational field. These perturbations are sound waves in normal fluids [1,2] and quasiparticles in superfluids [3] (phonons in superfluid ^{4}He and low-energy Bogoliubov fermions in superfluid ^{3}He-A). If the fluid motion is radial and spherically symmetric the effective metric is expressed in terms of the radial velocity $v(r)$ as:

$$ds^2 = -\left(c^2 - v^2(r)\right) dt^2 + 2v(r)drdt + dr^2 + r^2 d\Omega^2 \ . \tag{1}$$

For superfluids v is the velocity of superfluid vacuum v_s.

The kinetic energy of superflow plays the part of the gravitational potential: $\Phi = -v^2(r)/2$. If one chooses the velocity field corresponding to the potential of the point body of mass M

$$v^2(r) = -2\Phi = \frac{2GM}{r} \equiv c^2 \frac{r_h}{r} \ , \tag{2}$$

one obtains the Panlevé-Gullstrand form of Schwarzschild geometry (see e.g. ref. [2]). Here r_h denotes the position of the event horizon, where the velocity reaches the "speed of light" (c is the speed of sound for phonons or the slope in the energy spectrum $E = \pm cp$ of Bogoliubov fermions). If the fluid moves towards the origin, i.e. $v(r) < 0$, this velocity field reproduces the horizon of the black hole (the so called sonic black hole [1]): Since the velocity of the fluid behind horizon exceeds the speed c of the propagation of the perturbations with respect to the fluid, the low-energy quasipartilcles are trapped within the horizon. In quantum fermi liquids – superfluid phases of ^{3}He – such kind of the hydrodynamic black hole will allow us to investigate the quantum fermionic vacuum in a classical gravitational field in the presence of a horizon.

The hydrodynamic black hole was first suggested by Unruh for ordinary liquid [1]. However since all the known normal liquids are classical, the most interesting quantum effects related to the horizon cannot be simulated in such flow. Also the geometry is such that it cannot be realized: in the radial flow inward the liquid becomes accumulated at the origin, that is why this sonic black hole cannot be stationary. In the other scenario a horizon appears in moving solitons, if the velocity of the soliton exceeds the local "speed of light" [3]. This scenario has the same drawback: in finite system the motion of the soliton cannot be supported for a long time. In a draining bathtub geometry suggested in Ref. [4] the fluid motion can be made constant in time. However the friction of the liquid, which moves through the drain, is the main source of dissipation. This will hide any quantum effects related to the horizon. The superfluidity of the liquid does not help much in this situation: Horizon does not appear: The "superluminal" (supercritical) motion with respect to the boundaries is unstable, because the interaction with the walls produces the Cherenkov radiation of quasiparticles, and superfluidity collapses (see [5]).

Here we suggest a scenario, in which this collapse is avoided. The superfluid motion becomes quasi-stationary and exhibits the event horizon; the life time of the "superluminally" flowing state is determined by intrinsic mechanisms related to existence of a horizon, in particular by the analogue of Hawking radiation.

B. *Simulation of 2D black hole*

The stationary black hole can be realized in the following geometry, which is the development of the bathtub geometry of Ref. [4] (see Fig. 1(a)). The superfluid ^{3}He-A film is moving towards the center of the disk (i.e. $v(r) < 0$), where it escapes to the third dimension due to the orifice (hole). If the thickness of the film is constant, the flow velocity of the 2D motion increases towards the center as $v(r) = a/r$ and at $r = r_h = a/c$ it reaches the "speed of light" c (now r denotes the radial coordinate in the cylindrical system). If this happens the hole becomes the black hole: Behind the horizon, at $r < r_h$, the (quasi)particles can move only to the hole (orifice), since their velocity of propagation with respect to the superfluid condensate is less than the velocity v of the condensate.

The black hole analogy is also supported by the effective metric experienced by the quasiparticles. The energy spectrum of the low-energy Bogoliubov fermions is given by

$$(E - \mathbf{p} \cdot \mathbf{v})^2 = c^2(p_x^2 + p_y^2) + v_F^2(p_z \mp p_F)^2 \ . \tag{3}$$

Here the axis z is along the normal of the film. This axis z also marks the direction of the unit orbital vector $\hat{l}$, which is the anisotropy axis for the "speed of light": $\hat{l}$ is fixed along the normal to the film. The "speed of light" for quasiparticles propagating along the film is $c \sim 3$ cm/sec. It is much smaller than the Fermi velocity v_F which corresponds to the "speed of light" for quasiparticles propagating along the normal to the film. This c is also much smaller than the speed of sound in ^{3}He-A, that is why the motion of fluid has no effect on the density of the liquid.

Outside the orifice the velocity of the superfluid $\mathbf{v}$ is two dimensional and radial. With such velocity field the energy spectrum in Eq.(3) corresponds to the motion of the Bogoliubov quasiparticle in the space with following effective metric

$$ds^2 = -\left(c^2 - v^2(r)\right) dt^2 + 2v(r)drdt + dr^2 + r^2 d\phi^2 + \frac{c^2}{v_F^2} dz^2 \ . \tag{4}$$

Across the horizon the g_{00} component of the metric changes sign, which marks the presence of the horizon at $r = r_h$, where $v(r_h) = c$.

The important element of the construction in Fig. 1(a) is that the moving superfluid ^{3}He-A film is placed on the top of the superfluid ^{4}He film. This is made to avoid the interaction of the ^{3}He-A film with the solid substrate. The superfluid ^{4}He film effectively screens the interaction and thus prevents the collapse of the "superluminal" flow of ^{3}He-A.

The motion of the superfluid ^{3}He-A with respect to superfluid ^{4}He film is not dangerous: The superfluid ^{4}He is not excited even if ^{3}He-A moves with its superluminal velocity: c for ^{3}He-A is much smaller than the Landau velocity for radiation of quasiparticles in superfluid ^{4}He, which is about 50 m/sec. In this consideration we neglected the radiation of surface waves, assuming that the thickness of ^{4}He film is small enough.

Finally one can close the superflow by introducing the toroidal geometry. Fig. 1(b) shows the superflow around meridians (minor circles) of the torus in the cross-sectional plane. Both superfluid condensates, ^{4}He and ^{3}He-A, circulate around meridians with integer numbers N_4 and N_3 of superfluid velocity circulation quanta, $\kappa_4 = 2\pi\hbar/m_4$ and $\kappa_3 = \pi\hbar/m_3$. If the inner radius of the torus is small, the superfluid velocity is enhanced in the region close to the inner circle, where it can exceed c. In this case both the black hole horizon and the white hole horizon appear.

Since the extrinsic mechanism of the friction of ^{3}He-A film – the scattering of quasiparticles on the roughness of substrate – is abandoned, we can consider now intrinsic mechanisms of dissipation of the supecritical flow. The most interesting one is the Hawking radiation related to existence of a horizon.

C. *Vacuum in comoving and rest frames.*

Let us consider the simplest case of the 2D motion along the film in the bathtub geometry of Fig. 1(a). This can be easily generalized to the motion in the torus geometry.

There are two important reference frames: (i) The frame of the observer, who is locally comoving with the superfluid vacuum. In this frame the local superfluid velocity is zero, $v = 0$, so that the energy spectrum of the Bogoliubov fermions in the place of the observer is (here we assume a pure 2D motion along the film)

$$E_{\rm com} = \pm cp \ . \tag{5}$$

In this geometry, in which the superflow velocity is confined in the plane of the film, the speed c coincides with the Landau critical velocity of the superfluid vacuum, $v_{\rm Landau} = \min(|E_{\rm com}(p)|/p)$. The vacuum as determined by the

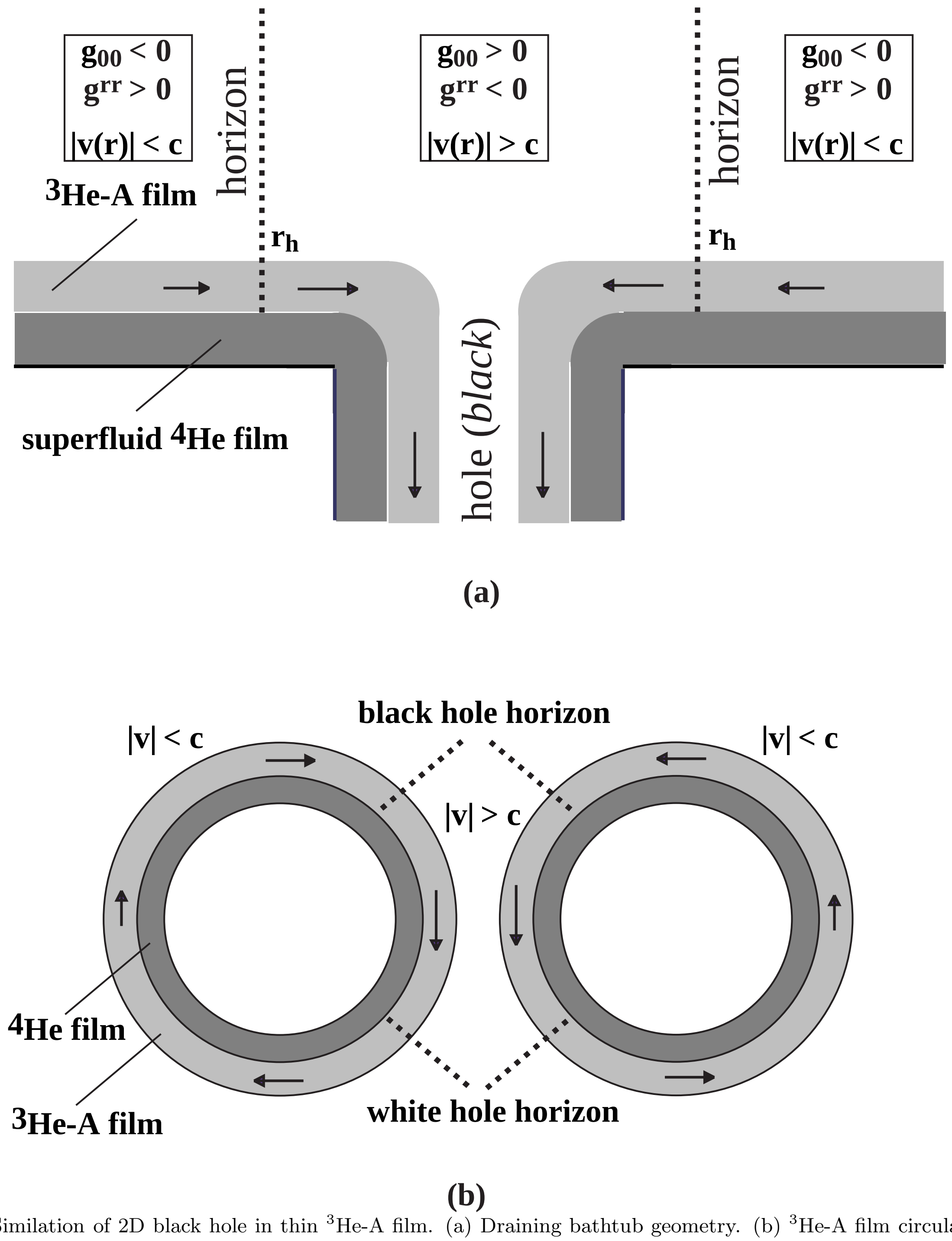

FIG. 1. Similation of 2D black hole in thin ^{3}He-A film. (a) Draining bathtub geometry. (b) ^{3}He-A film circulating on the top of the ^{4}He film on torus.

comoving observer is shown on Fig. 2(a): fermions occupy the negative energy levels in the Dirac sea (the states with the minus sign in Eq.(5). It is the counterpart of the Minkowski vacuum, which is however determined only locally: The comoving frame cannot be determined globally. Moreover for the comowing observer, whose velocity changes with time, the whole velocity field $\mathbf{v}(\mathbf{r},t)$ of the superflow is time dependent. This does not allow to determine the energy correctly.

(ii) The energy can be well defined in the laboratory frame (the rest frame). In this frame the system is stationary,

though is not static: The effective metric does not depend on time, so that the energy is conserved, but this metric contains the mixed component g_{0i}. The energy in the rest frame is obtained from the local energy in the comoving frame by the Doppler shift. In case of the radial superflow one has

$$E_{\rm rest} = \pm cp + p_r v(r) \ . \tag{6}$$

Figs. 2(b-c) show how the "Minkowski" vacuum of the comoving frame is seen by the rest observer (note that the velocity is negative, $v(r) < 0$). In the absense of horizon, or outside the horizon the local vacuum does not change: the states which are occupied (empty) in the Minkowski vacuum remain occupied (empty) in the rest frame vacuum (see Fig. 2(b)). In the presense of horizon behind which the velocity of superflow exceeds the Landau critical velocity the situation changes: Behind the horizon the vacuum in the rest frame differs from that in the comoving frame. Let us for simplicity consider the states with zero transverse momentum $p_\phi = 0$ on the branch $E_{\rm rest} = (v(r) + c)p_r$ in the rest frame. If the system is in the Minkowski vacuum state (i.e. in the ground state as viewed by comoving observer), quasiparticles on this branch has reversed distribution in the rest frame: the negative energy states are empty, while the positive energy states are occupied (see Fig. 2(c)). For this branch the particle distribution corresponds to the negative temperature $T = -0$ behind horizon.

Since the energy in the rest frame is a good quantum number, the fermions can tunnel across the horizon from the occupied levels to the empty ones with the same energy. Thus if the system is initially in the Minkowski vacuum in the comoving frame, the tunneling disturbs this vacuum state: Pairs of excitations are created: the quasiparticle, say, is created outside the horizon while its partner – the quasihole – is created inside the horizon. This simulates the Hawking radiation from the black hole.

D. *Hawking radiation*

To estimate the tunneling rate in the semiclassical approximation, let us consider the classical trajectories $p_r(r)$ of particles, say, with positive energy, $E_{\rm rest} > 0$, for the simplest case when the transverse momentum p_ϕ is zero Fig. 3. The branch $E_{\rm rest} = (v(r) - c)p_r$ describes the incoming particles with $p_r < 0$ which propagate through the horizon to the orifice (or to the singularity at $r = 0$, if the orifice is infinitely small) without any singularity at the horizon. The classical trajectories of these particles are

$$p_r(r) = -\frac{E_{\rm rest}}{c - v(r)} < 0 \ . \tag{7}$$

The energies of these particles viewed by the comoving observer are also positive: $E_{\rm com}(r) = -cp_r(r) = E_{\rm rest}(c/c - v(r)) > 0$.

Another branch $E_{\rm rest} = (v(r) + c)p_r$ in Fig. 3 contains two disconnected pieces describing the particle propagating from the horizon in two opposite directions:

$$r > r_h \ : \ p_r(r) = \frac{E_{\rm rest}}{c + v(r)} > 0 \ , \ E_{\rm com}(r) = cp_r(r) > 0 \ . \tag{8}$$

$$r < r_h \ : \ p_r(r) = \frac{E_{\rm rest}}{c + v(r)} < 0 \ , \ E_{\rm com}(r) = cp_r(r) < 0 \ , \tag{9}$$

The Eq.(8) describes the outgoing particles – the particles propagating from the horizon to the exterior. The energy of these particles is positive in both frames, comoving and rest. The Eq.(9) describes the propagation of particles from the horizon to the orifice (or to the singularity). Though for the rest frame observer the energy of these particles is positive, these particles, which live within the horizon, belong to the Minkowski vacuum in the comoving frame.

The classical trajectory in Eqs.(8,9) is thus disrupted at the horizon. There is however a quantum mechanical transition between the two pieces of the branch: the quantum tunneling. The tunneling amplitude can be found in semiclassical approximation by shifting the contour of integration to the complex plane:

$$w \sim \exp(-2S) \ , \tag{10}$$

$$S = \mathbf{Im} \int dr \ p_r(r) = \frac{\pi E_{\rm rest}}{|v'(r)|_{r=r_h}} \ . \tag{11}$$

This means that the wave function of any particle in the Minkowskii vacuum inside the horizon contains an exponentially small part describing the propagation from the horizon to infinity. This corresponds to the radiation from

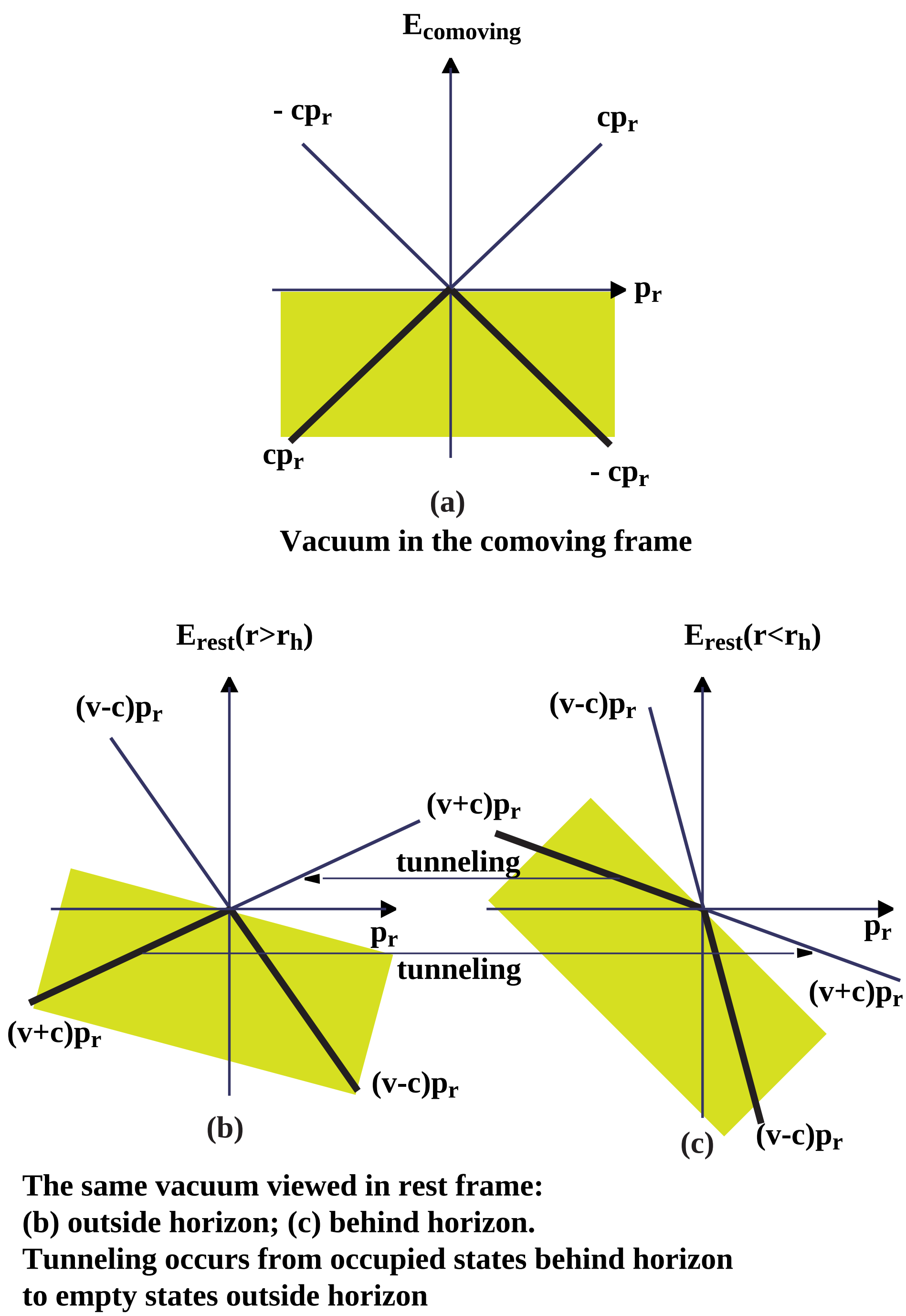

FIG. 2. (a) Fermionic vacuum in the comoving frame. The states with $E_{\rm com} < 0$ are occupied (thick lines). The same vacuum viewed in the rest frame (b) outside horizon and (c) inside horizon. Behind the horizon the branch $E_{\rm rest} = (v + c)p_r$ (for $p_\perp = 0$) has inverse population as seen in the rest frame: the states with positive enegry $E_{\rm rest} > 0$ are filled, while the states with $E_{\rm rest} < 0$ are empty. The tunneling across horizon from the occupied states to the empty states with the same energy gives rise to the Hawking radiation from the horizon.

the Minkowski vacuum in the presence of the event horizon. The exponential dependence of the probability on the

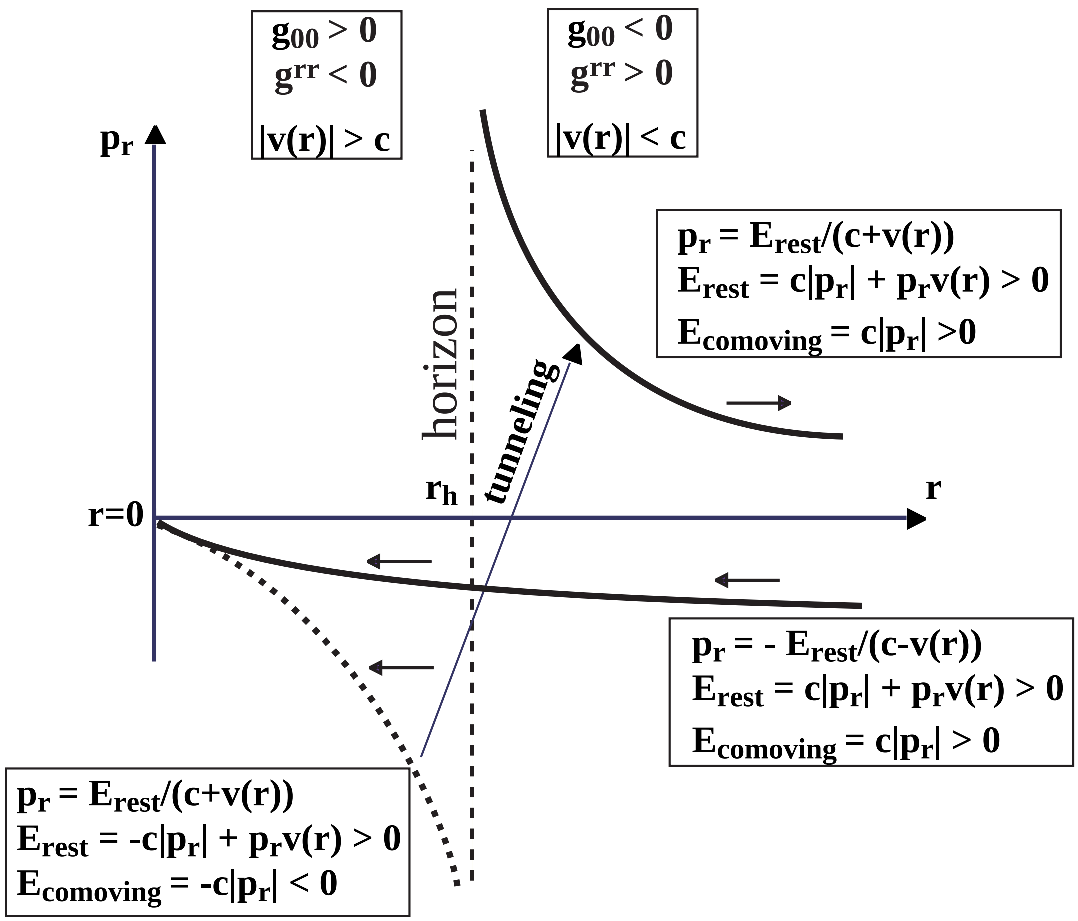

FIG. 3. Tunneling from Minkowski vacuum within the horizon to the outgoing mode.

quasiparticle energy $E_{\rm rest}$ suggests that this radiation looks as thermal. The corresponding temperature, the Hawking temperature, is

$$T_{\rm Hawking} = \frac{\hbar |v'(r)|_{r=r_h}}{2\pi} \ . \tag{12}$$

The radiation leads to the quantum friction: the linear momentum of the flow decreases with time. This occurs continuously until the superfluid Minkowski vacuum between the horizons is completely exhausted and the superfluid state is violated. This leads to the phase slip event, after which the number N_3 of circulation quanta of superfluid velocity trapped by the torus is reduced. This process will repeatedly continue until the two horizons merge.

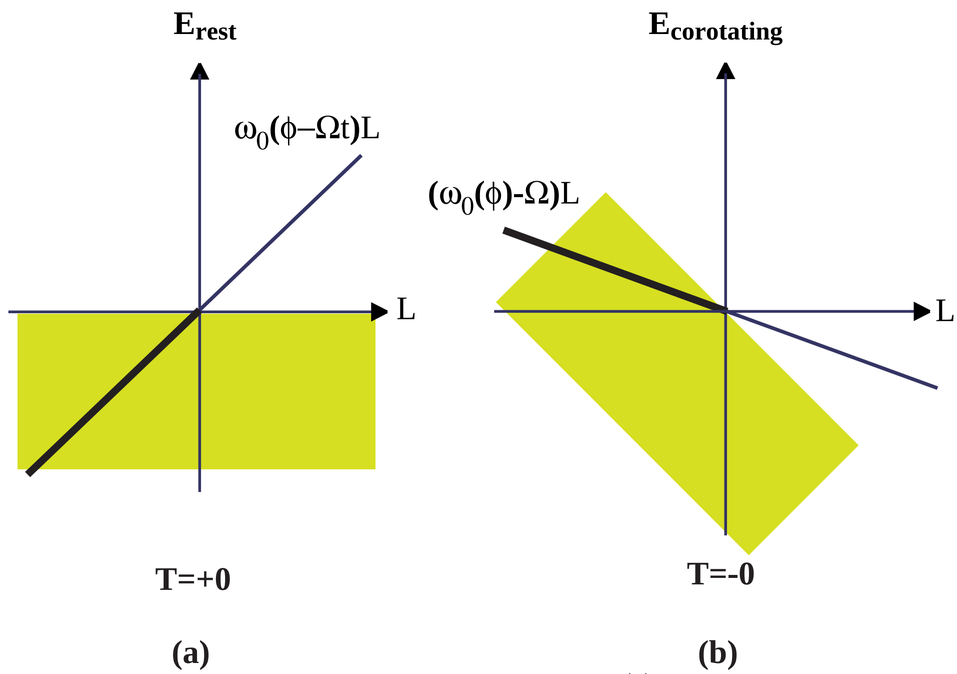

FIG. 4. Enegry spectrum of fermionic quasiparticles in the vortex core. (a) $\Omega = 0$ corresponds to the nonrotating vortex. The spectrum is linear in the angular momentum L; $\omega_0(\phi)$ is the minigap, which depends on the angle ϕ if the core is not axisymmetric. The states with negative L are occupied. If the vortex core is rotating with angular velocity Ω, then in the laboratory frame the minigap depends on time. (b) The spectrum is well defined in the coordinate frame corotating with the core. Here the spectrum is shown in the region of ϕ where $\omega_0(\phi) < \Omega$, i.e. behind the horizon. If the initial state is the Minkowski vacuum, it is seen by the corotating observer as the state with $T = -0$.

E. *Negative temperature for the chiral 1+1 fermions.*

It should be mentioned that there is an example where the negative temperature behind a horizon is well defined. This is the case of the 1+1 dimensional chiral fermions living in the vortex core. For such fermions there is only one branch, $E = \omega_0(\phi)L$. Here L is the angular momentum of the quasiparticle in the vortex core; $\omega_0(\phi)$ is the so-called minigap, which for the nonaxisymmetric vortex core depends on the azimuthal angle ϕ. This is equivalent to our branch $E = c(r)p_r$ in the nonmoving liquid if the speed of light is coordinate dependent. If the vortex core is rotated with angular velocity Ω, the energy spectrum is time dependent in the laboratory frame $E = \omega_0(\phi - \Omega t)L$. But it is time independent in the frame corotating with the vortex core, where the energy is well determined: $E_{\rm corotating} = (\omega_0(\phi) - \Omega)L$; this is equivalent to our branch $E_{\rm rest} = (v + c)p_r$ in the rest frame. The horizon can occur if the vortex core rotates with sufficently large angular velocity, such that Ω exceeds the minimal value of the minigap [6]. In this case Since there is only one branch of the fermions the negative temperature is well determined. Behind the horizon the Minkowski vacuum, which is the state with $T = +0$ now in the laboratory frame Fig. 4(a), is really the state with $T = -0$ in the frame corotating with the core Fig. 4(b); and vice versa, the state with $T = -0$ in the laboratory frame is the state with $T = +0$ in the corotating frame.

Such symmetry between the vacua for the 1+1 chiral fermions suggests that there can be also the symmetry between the nonzero positive and negative temperatures. Let us now take into account the Hawking radiation and suppose that at infinity there is a heat bath with the temperature $T = T_{\rm Hawking}$. Then the heat flux from infinity exactly compensates the radiation from the horizon. In such a metastable steady state the distribution of quasiparticles behind horizon would correspond to a nonzero negative temperature $T = -T_{\rm Hawking}$.

F. *Discussion.*

The above construction in Fig. 1(b) allows us (at least in principle) to obtain the event horizon in the quasi-stationary regime, when the main source of nonstationarity is the dissipation coming from the Hawking radiation. As

for the practical realization, there are, of course, many technical problems to be solved. On the other hand, if the black hole analog can in principle exist in condensed matter as the quasi-stationary object, its prototype – the real black hole – can also exist, at least in principle (though it is not so easy to find the scenario of how this object can be obtained from the gravitational collapse of matter [7]).

The situation, in which the supercritical flow is described in terms of the event horizon and Hawking radiation, occurs only for the low-energy fermions, whose spectrum is "relativistic". Howeverever, the analogue of event horizon persists even in the case of the "nonrelativistic" spectrum: the "horizon" occurs at the surface where the local superflow becomes supercritical, i.e. the superfluid velocity exceeds the Landau critical velocity. So it is necessary to extend the consideration of the Hawking type radiation to higher energies, where the other mechanisms of the decay of the supercritical superflow can become important. For example, the radiated particles with energies outside the "relativistic" region can be Andreev scattered back to the black hole. Thus both partners (particle and hole) of the Hawking radiation will remain within the horizon. This means that the particle creation in high gravity field can disturb the Minkowski quantum vacuum inside the horizon without any radiation to the exterior. In principle such pair creation inside the horizon can be more important for the dissipation of the "superluminal" (supercritical) superflow than the Hawking radiation.

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